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Convective instability of self-similar spherical expansion into a vacuum

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The well-known class of self-similar solutions for an ideal polytropic gas sphere of radius $R(t)$ expanding into a vacuum with velocity $u(r,t) = r\dot{R}/R$ is shown to be convectively unstable. The physical mechanism results from the buoyancy force experienced by anisentropic distributions in the inertial (effective gravitational) field. An equation for the perturbed displacement $\boldsymbol{\xi}(\mathbf{r},t)$, derived from the linearized fluid equations in Lagrangian co-ordinates, is solved by separation of variables. Because the basic state is non-steady, the perturbations do not grow exponentially, but can be expressed in terms of hypergeometric functions. For initial density profiles

$$\rho_0(r) \sim (1 - r^2/r_0^2)^\kappa,$$

modes with angular dependence $Y_{lm}(\theta,\phi)$ are unstable provided $l > 0$ and $\kappa < 1/(\gamma-1)$, where γ is the ratio of specific heats. For large l, the characteristic growth time of the perturbations varies as $l^{-\frac{1}{2}}$ and the amplification increases exponentially as a function of l. The radial eigenfunctions are proportional to r^l, and the compressibility and vorticity are both non-zero.

1. Introduction

Over a quarter century has passed since the discovery of a variety of self-similar solutions of the equations of one-dimensional ideal fluid motion, describing non-steady expansion and contraction. These were found independently and more or less simultaneously by Sedov (1953), Staniukovich (1949), Taylor (1950) and others, stimulated by interest in nuclear and astrophysical explosions and in the general properties of gasdynamic systems. The usefulness of these solutions is twofold: they correctly describe one-dimensional flows at late times when the details of initiation or preparation have been 'forgotten', and they are analytic, or at least reduce the solution to quadratures.

A particularly useful and interesting type of self-similar motion is that known as uniform or homogeneous. Its characteristic feature is a radial velocity which is proportional to the distance from the centre of symmetry. Most applications have been to problems with spherical symmetry, such as supernova explosions (Keller 1956), laser implosions (Kidder 1976) and self-gravitating clouds (Sedov 1959). Closely related to the latter are cosmological models in the non-relativistic limit (Weinberg 1972), which are distinctive by virtue of being pressureless and unbounded.

Sedov (1959) distinguishes three types of uniform self-similar motion in an ideal

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gas. In type I, the radius varies between 0 and ∞. In type II, it varies between 0 and a finite maximum value, corresponding to a turning point. Type III, to which we restrict ourselves in the present work, has radius varying between a minimum (at a turning point) and ∞. We follow Keller (1956) in using Lagrangian co-ordinates to derive in § 2 a two-parameter family of solutions including, among others, isothermal and uniform-density models as special cases. There seems at first glance no reason why these should not be stable, and indeed in the literature where applications are made (e.g. Zel'dovich & Raizer 1966) the possibility has apparently not been considered before. We go on in § 3, however, to argue that instability should arise whenever a condition is satisfied, equivalent to the presence of an entropy density which *decreases* outward. Physically the mechanism is identical with that responsible for convective instability in static stratified media when the temperature decreases in the upward direction. Analysis of the linearized fluid equations in § 4 using the techniques developed by Bernstein & Book (1978) and Book (1978) confirms the existence of the instability and yields both the space and time dependence of the perturbations in closed form.

As was previously noted by Bernstein & Book (1978) and Book & Bernstein (1979), the usual definition of stability is inadequate when applied to non-steady states, since the time dependence of the perturbations is in general not exponential. It is appropriate to call a mode stable (unstable) if the relative amplification, i.e. the ratio of the perturbation amplitude to that of the basic state, vanishes (diverges) as $t \to \infty$. We find that in the present case the ratio is in general finite because the acceleration responsible for the growth of 'unstable' perturbations vanishes as $|t| \to \infty$. Thus the system is effectively destabilized only for a finite time. However, the relative amplification can be made arbitrarily large in unstable systems by choice of sufficiently large mode number.

The paper concludes in § 5 with a brief discussion of the results.

2. The basic state

We start with the equations of ideal hydrodynamics which in Lagrangian variables take the form

$$\dot{\rho} + \mathbf{v} \,.\, \nabla \rho = 0, \tag{2.1a}$$

$$\rho \dot{\mathbf{v}} + \nabla p = 0 \tag{2.1b}$$

and

$$(p\rho^{-\gamma})^{\cdot} = 0. \tag{2.1c}$$

Here dots denote time derivatives and γ in (2.1 c) is the ratio of specific heats.

In a spherically symmetric system, (2.1 a, b) become

$$\dot{\rho} + \rho R^{-2} \frac{\partial}{\partial R}(R^2 u) = 0 \tag{2.2a}$$

and

$$\rho \dot{u} + \frac{\partial p}{\partial R} = 0. \tag{2.2b}$$

For a motion of the type known as homogeneous (or uniform) self-similar flow (Sedov 1959), the position R at time t of a fluid element whose position at $t = 0$ was r is required to satisfy

$$R = rf(t), \tag{2.3}$$

where $f(0) = 1$ and $\dot{f}(0) = 0$. The continuity equation (2.2*a*) then yields

$$\rho(r,t) = \rho_0(r) f^{-3}, \tag{2.4}$$

and hence from the adiabatic law (2.1*c*)

$$p(r,t) = p_0(r) f^{-3\gamma} = s(r)\rho_0^\gamma f^{-3\gamma}, \tag{2.5}$$

with the entropy function s arbitrary. We choose the initial density profile in the form

$$\rho_0(r) = \hat{\rho}(1 - r^2/r_0^2)^\kappa, \tag{2.6}$$

where $\hat{\rho}$ and κ are constants. The density is then uniform for $\kappa = 0$, and becomes more and more strongly peaked at $r = 0$ as κ increases. It follows from (2.1*b*) that

$$p_0(r) = \hat{p}(1 - r^2/r_0^2)^{\kappa+1}, \tag{2.7}$$

and f must satisfy

$$\ddot{f} f^{3\gamma-2} = 2(\kappa+1)\hat{p}/\hat{\rho} r_0^2 = \tau^{-2}. \tag{2.8}$$

We will use the separation constant τ, the initial radius r_0, and the peak mass density $\hat{\rho}$ to rescale t, r and ρ, respectively. In these reduced variables we have

$$\rho_0 = (1 - r^2)^\kappa, \tag{2.9}$$

$$p_0 = (1 - r^2)^{\kappa+1}/2(\kappa+1) \tag{2.10}$$

and

$$\ddot{f} f^{3\gamma-2} = 1. \tag{2.11}$$

A quadrature can be performed on (2.11), with the result

$$\dot{f}^2 = 2\ln f \tag{2.12}$$

if $\gamma = 1$ (the isothermal case), and

$$\dot{f}^2 = (2/\alpha)(1 - f^{-\alpha}) \tag{2.13}$$

otherwise, where $\alpha = 3(\gamma - 1)$. If $\gamma = \frac{5}{3}$, (2.13) can be integrated directly to give $f(t) = \pm(1 + t^2)^{\frac{1}{2}}$. For other values of γ the solution is most conveniently found by numerical means. At large $|t|$ when $\ddot{f} \to 0$, the motion asymptotically approaches free streaming. As a function of the parameters κ and γ, the solutions include the cases of uniform density and quadratic pressure, $\kappa = 0$, and uniform entropy density, $\kappa = 1/(\gamma - 1)$. If we exclude singular density profiles, κ is restricted to $0 \leqslant \kappa < \infty$. The other parameter is γ, which must lie in the range $1 \leqslant \gamma < \infty$.

3. Physical mechanism for instability

At time t a small volume ΔV of fluid initially located at radius r contains a mass $\Delta m = \rho_0(r)\Delta V f^{-3}$, subjected to a pressure $p = (1 - r^2)^{\kappa+1}/[2(\kappa+1) f^{3\gamma}]$. Consider two such fluid elements initially at radii r_1 and $r_2 > r_1$, whose volumes are related by

$$\Delta V_2/\Delta V_1 = (p_1/p_2)^{1/\gamma} = [(1 - r_1^2)/(1 - r_2^2)]^{(\kappa+1)/\gamma}. \tag{3.1}$$

This choice is made so that, after adiabatically interchanging positions, the two elements will be in pressure balance with the surrounding medium. The compressional energy associated with these elements is

$$W_p = (p_1 \Delta V_1 + p_2 \Delta V_2)/(\gamma - 1). \tag{3.2}$$

Their kinetic energy calculated from the expansion or contraction of the sphere is

$$W_k = \tfrac{1}{2}(\Delta m_1 u_1^2 + \Delta m_2 u_2^2). \tag{3.3}$$

Because the state is non-steady ($\ddot{f} \neq 0$), the elements are subject to an effective inertial force, derivable from a potential. The local acceleration in the laboratory frame is $g = r\ddot{f} = R\ddot{f}/f$, and therefore the effective potential energy can be written

$$W_g = \frac{1}{2}\frac{\ddot{f}}{f}(\Delta m_1 r_1^2 + \Delta m_2 r_2^2). \tag{3.4}$$

Now let the two fluid elements interchange positions. By (3.1), they contract or expand so as to satisfy local pressure balance after the interchange. Furthermore, the work done in compressing one is just balanced by that done by the expansion of the other, so the compressional energy W'_p afterwards is equal to W_p. The net change in energy is then

$$\begin{aligned}\delta W &= W'_p + W'_k + W'_g - W_p - W_k - W_g \\ &= \tfrac{1}{2}[\Delta m_1 u_2^2 + \Delta m_2 u_1^2 - \Delta m_1 u_1^2 - \Delta m_2 u_2^2] + \tfrac{1}{2} f\ddot{f}[\Delta m_1 r_2^2 + \Delta m_2 r_1^2 - \Delta m_1 r_1^2 - \Delta m_2 r_2^2] \\ &= \tfrac{1}{2}(\dot{f}^2 + f\ddot{f})(r_2^2 - r_1^2)(\Delta m_1 - \Delta m_2). \end{aligned} \tag{3.5}$$

The first and second factors in the last member of (3.5) are strictly positive. The third factor, on the other hand, is proportional to

$$(1-r_1^2)^\kappa - (1-r_2^2)^\kappa \left(\frac{1-r_1^2}{1-r_2^2}\right)^{(\kappa+1)/\gamma} = (1-r_1^2)\left[1 - \left(\frac{1-r_1^2}{1-r_2^2}\right)^{(\kappa+1-\kappa\gamma)/\gamma}\right]. \tag{3.6}$$

This expression is negative for $\kappa < 1/(\gamma-1)$. In this case, therefore, the interchange *reduces* the total system energy. We thus anticipate that an instability will set in, characterized by 'overturning' of the profiles, such as is typically seen in convective or thermal instabilities of static media (Landau & Lifshitz 1959).

When $\kappa < 1/(\gamma-1)$, $\delta W > 0$, in which case no instability should arise. By (2.5), the entropy function s satisfies

$$s(r) = (1-r^2)^{\kappa+1-\kappa\gamma}. \tag{3.7}$$

The stable (unstable) case corresponds to outward increasing (decreasing) $s(r)$. The marginal case just corresponds to isentropic – more properly, homentropic – states. Evidently the physical picture here is analogous to that arising in connexion with instabilities driven by a temperature inversion in a medium with a stratified density. Destabilization takes place owing to the buoyancy experienced by fluid elements in the non-uniform inertial gravity field. It is therefore purely a consequence of the non-steady character of the basic state.

4. Analysis of the perturbed equations

We follow Bernstein & Book (1978) and Book (1978) in obtaining linearized equations for the development of a small perturbation about the solutions of § 2. For simplicity we consider only expanding states ($t > 0$). The perturbed displacement $\boldsymbol{\xi}$ satisfies the linearized form of (2.1*b*),

$$\rho\ddot{\boldsymbol{\xi}} + \rho_1 \ddot{\mathbf{R}} = -\nabla_{\mathbf{R}} p_1 + \nabla_{\mathbf{R}}\boldsymbol{\xi} \,.\, \nabla_{\mathbf{R}} p. \tag{4.1}$$

Substituting the perturbed density from

$$\rho_1 = -\rho \nabla_{\mathbf{R}} . \boldsymbol{\xi}, \tag{4.2}$$

and the perturbed pressure from

$$p_1 = \frac{\partial p}{\partial \rho} \rho_1 = -\frac{\gamma(1-r^2)^{1+\kappa(1-\gamma)}}{2(\kappa+1)} \rho^{\gamma} \nabla_{\mathbf{R}} . \boldsymbol{\xi}, \tag{4.3}$$

and noting that $\mathbf{R}$ and $\mathbf{r}$ are related by (2.3), we obtain (writing $\nabla = \nabla_{\mathbf{r}}$)

$$f^{\alpha+2} \ddot{\boldsymbol{\xi}} = \frac{\gamma(1-r^2)}{2(\kappa+1)} \nabla(\nabla . \boldsymbol{\xi}) - (\gamma-1)\, \mathbf{r} \nabla . \boldsymbol{\xi} - \nabla \boldsymbol{\xi} . \mathbf{r}. \tag{4.4}$$

Letting $\sigma = \nabla . \boldsymbol{\xi}$ and $\boldsymbol{\omega} = \nabla \times \boldsymbol{\xi}$, we have, on taking the divergence and curl of (4.4),

$$f^{\alpha+2} \ddot{\sigma} = \frac{\gamma}{2(\kappa+1)} \nabla . [(1-r^2) \nabla \sigma] - \gamma \mathbf{r} . \nabla \sigma - (3\gamma - 2) \sigma + \mathbf{r} . \nabla \times \boldsymbol{\omega} \tag{4.5}$$

and

$$f^{\alpha+2} \ddot{\boldsymbol{\omega}} = \left(1 - \frac{\gamma\kappa}{\kappa+1}\right) \nabla \sigma \times \mathbf{r} + \boldsymbol{\omega}. \tag{4.6}$$

We look for solutions of (4.5)–(4.6), assuming $\boldsymbol{\xi}$ is separable into a product of a function of position and a factor $T(t)$ satisfying

$$f^{\alpha+2} \ddot{T} = \mu T, \tag{4.7}$$

μ constant. We further assume separation of the angular and radial dependence by writing

$$\sigma(\mathbf{r}) = \sigma(r) Y_{lm}(\theta, \phi). \tag{4.8}$$

In (4.5) $\boldsymbol{\omega}$ appears only in the form $\mathbf{r} . \nabla \times \boldsymbol{\omega}$, for which an expression in terms of σ can be derived from (4.6) and (4.8):

$$\begin{aligned}(\mu - 1)\, \mathbf{r} . \nabla \times \boldsymbol{\omega} &= \frac{1-(\gamma-1)\kappa}{\kappa+1} [2\mathbf{r} . \nabla \sigma + \mathbf{r}\mathbf{r} : \nabla\nabla\sigma - r^2 \nabla^2 \sigma] \\ &= \sigma[1-(\gamma-1)\kappa]\, l(l+1)/(\kappa+1). \end{aligned} \tag{4.9}$$

Substitution in (4.5) yields a second-order equation for the radial factor $\sigma(r)$,

$$\frac{\gamma}{2(\kappa+1)} \left[\frac{1-r^2}{r^2} \frac{d}{dr}\left(r^2 \frac{d\sigma}{dr}\right) - \frac{1-r^2}{r^2} l(l+1)\, \sigma - 2r \frac{d\sigma}{dr} \right] - \gamma r \frac{d\sigma}{dr} + \left[\frac{(\kappa+1-\kappa\gamma)\, l(l+1)}{(\mu-1)(\kappa+1)} - 3\gamma - \mu + 2 \right] \sigma = 0. \tag{4.10}$$

Rewriting this equation by means of the substitutions $\sigma = r^l y$ and $x = r^2$, we obtain the hypergeometric equation

$$x(1-x)\, y'' + [c - (a+b+1)\, x]\, y' - aby = 0, \tag{4.11}$$

where

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \tfrac{1}{2}\{\kappa + l + \tfrac{5}{2} \pm [(\kappa + l + \tfrac{5}{2})^2 - 4K]^{\frac{1}{2}}\}, \tag{4.12a, b}$$

$$c = l + \tfrac{3}{2}. \tag{4.12c}$$

Here

$$K = \frac{(\kappa+2)\, l}{2} + \frac{\kappa+1}{2\gamma} \left[\mu - 2 + 3\gamma - \frac{l(l+1)(\kappa+1-\kappa\gamma)}{(\mu-1)(\kappa+1)} \right]. \tag{4.13}$$

The solution of (4.11) which is finite at the origin is the hypergeometric function $y = {}_2F_1(a, b; c; x)$.

The boundary condition is found from the requirement that the perturbed pressure vanish on the interface with the vacuum. Since the unperturbed pressure already vanishes there, it follows from (3.3) that y need only be finite at $x = 1$. The linear connexion formulas (e.g. Abramowitz & Stegun 1964) contain a term that diverges as $(1-x)^{-(\kappa+1)}$ unless a or b is a non-positive integer. Thus we must have

$$-n = \tfrac{1}{2}\{\kappa+l+\tfrac{5}{2}-[(\kappa+l+\tfrac{5}{2})^2-4K]^{\frac{1}{2}}\}, \tag{4.14}$$

$n = 0, 1, 2, \ldots$. Treating n and K as analytic functions of μ and differentiating (4.14) yields $\partial n/\partial\mu < 0$. Hence the fastest growth (largest $\mu > 1$) corresponds to the *smallest* value of n, *viz.*, $n = 0$, which implies $K = 0$. Solving for μ, we finally obtain the dispersion relation

$$\mu-1 = -\frac{\gamma l(\kappa+2)+(\kappa+1)(3\gamma-1)}{2(\kappa+1)} \pm\frac{\{[\gamma l(\kappa+2)+(\kappa+1)(3\gamma-1)]^2+4l(l+1)(\kappa+1-\kappa\gamma)\}^{\frac{1}{2}}}{2(\kappa+1)}. \tag{4.15}$$

For the upper branch, $\mu > 1$ for all $l > 0$, provided $\kappa < 1/(\gamma-1)$. The latter is precisely the condition derived from the energetic argument of § 3.

Returning to (4.7), we find that, provided $\gamma > 1$, the time dependence can likewise be expressed in terms of hypergeometric functions in the form (Bernstein & Book 1978)

$$T(t) = T(0)\mathscr{F}(t)+\dot{T}(0)\mathscr{G}(t). \tag{4.16}$$

Here,

$$\mathscr{F}(t) = {}_2F_1\left[\frac{1}{4}+\frac{2+\Delta}{4\alpha}, \frac{1}{4}+\frac{2-\Delta}{4\alpha}; \frac{1}{2}; 1-f^{-\alpha}\right], \tag{4.17a}$$

$$\mathscr{G}(t) = \left[\frac{2}{\alpha}(1-f^{-\alpha})\right]^{\frac{1}{2}} {}_2F_1\left[\frac{3}{4}+\frac{2+\Delta}{4\alpha}, \frac{3}{4}+\frac{2-\Delta}{4\alpha}; \frac{3}{2}; 1-f^{-\alpha}\right], \tag{4.17b}$$

and $\Delta = [(\alpha+2)^2-8\mu\alpha]^{\frac{1}{2}}$. For late times (large f), the linear connexion formulas yield

$$\mathscr{F}(t) \sim \frac{\Gamma(\frac{1}{2})\,\Gamma(1/\alpha)f}{\Gamma[\frac{1}{4}+(2+\Delta)/4\alpha]\,\Gamma[\frac{1}{4}+(2-\Delta)/4\alpha]}, \tag{4.18a}$$

$$\mathscr{G}(t) \sim \frac{(2/\alpha)^{\frac{1}{2}}\,\Gamma(\frac{3}{2})\,\Gamma(1/\alpha)f}{\Gamma[\frac{3}{4}+(2+\Delta)/4\alpha]\,\Gamma[\frac{3}{4}+(2-\Delta)/4\alpha]}. \tag{4.18b}$$

The numerical coefficients in (4.18) grow exponentially with μ for $\mu \gg 1$. The case of $\gamma = 1$ is very similar, except that confluent hypergeometric functions replace ${}_2F_1$, as observed by Bernstein & Book (1978) and Book & Bernstein (1979), and (4.18) is replaced by expressions proportional to $f(\ln f)^{\frac{1}{2}(\mu-1)}$.

5. Discussion

We have seen on energetic grounds that a certain class of spherical ideal gas expansions can be expected to be unstable whenever the gradient of the entropy density decreases with increasing r. Detailed analysis of the linear perturbations about these non-steady basic states confirms this prediction, provided we appropriately generalize the usual definition of instability. Somewhat surprisingly, the solutions fall out exactly without recourse to numerical approximations, owing to the separability in Lagrangian variables of the linearized equations.

As noted in § 1, what matters in determining the stability of a time-dependent motion is the *relative* size of the perturbations. By (4.18), the latter vary asymptotically like the unperturbed radius. At early times, however, when $\ddot{f}$ differs substantially from zero, the perturbations can be amplified dramatically. If $\mu \gg 1$, they grow approximately exponentially for $t \lesssim 1$, experiencing $\sim \mu^{\frac{1}{2}}$ *e*-foldings. The total amplification and the time required to approach the asymptotic state both increase with μ. As $\gamma \rightarrow 1$, both the total amplification and the time required to approach saturation diverge (Bernstein & Book 1978). Since μ increases with increasing l, decreasing κ, and decreasing γ, all of these trends tend to enhance instability.

Note that, as $l \rightarrow \infty$, μ diverges. This implies that the problem is mathematically well posed only for sufficiently smooth initial perturbations. In any real physical system, however, dissipative phenomena related to viscosity, thermal conduction, radiation, etc. set an upper limit on the mode number for which the ideal fluid model is valid. For shorter-wavelength disturbances than this, not only the detailed perturbation analysis, but the whole physical picture must be drastically different.

The perturbations studied here have radial dependence which peaks at $r = r_0$. They therefore should be most readily observable as an enhanced mixing or turbulence near the periphery of the expanding cloud. Since the instability is controlled by the sign of the entropy gradient, it seems likely that the nonlinear limit to which it tends is characterized by $ds/dr \geqslant 0$, $0 \leqslant r \leqslant r_0$. Whether this limit is actually attained is beyond the scope of the present work.

Another, perhaps more important, question remains unanswered. Uniform self-similar motion is an analytically convenient model used to approximate real flows. To what extent is the instability treated here associated with the latter, to what extent an artifact of the model? The present paper can of course provide no rigorous answer. Nonetheless, it seems physically plausible that, for flows sufficiently close to uniform expansion, the results of the present analysis must be applicable. Even for non-uniform motions, either analytically or numerically described, the energetic argument of § 3 can be employed and should again correctly predict the presence or absence of instability.

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REFERENCES

Abramowitz, M. & Stegun, I. A. 1964 *Handbook of Mathematical Functions*. Washington: National Bureau of Standards.

Bernstein, I. B. & Book, D. L. 1978 *Astrophys. J.* **225**, 633.

Book, D. L. 1978 *Phys. Rev. Lett.* **41**, 1552.

Book, D. L. & Bernstein, I. B. 1979 *Phys. Fluids* **22**, 79.

Keller, J. B. 1956 *Quart. Appl. Math.* **14**, 171.

Kidder, R. E. 1976 *Nuclear Fusion* **16**, 3.

Landau, L. D. & Lifshitz, E. M. 1959 *Fluid Mechanics*, p. 8. Reading, MA.: Addison-Wesley.

Sedov, L. I. 1953 *Dokl. Akad. Nauk. S.S.S.R.* **90**, 753.

Sedov, L. I. 1959 *Similarity and Dimensional Methods in Mechanics*, pp. 271–281. New York: Academic Press.

Staniukovich, K. P. 1949 *Dokl. Akad. Nauk S.S.S.R.* **64**, 467.

Taylor, G. I. 1950 *Proc. Roy. Soc.* A **201**, 155.

Weinberg, S. 1972 *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, pp. 571–578. Wiley.

Zel'dovich, Ya. B. & Raizer, Yu. P. 1966 *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*, pp. 104–106. New York: Academic Press.